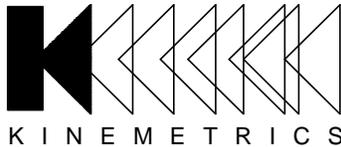


Application Note #39



Transfer Function of Kinematics Instruments

S-Plane Representations

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Transfer Functions of Kinematics Instruments

1. Introduction

The following report describes the s-plane transfer functions of Kinematics' instrumentation. Its purpose is to allow you to correct the instrument response where appropriate and contains the theoretical transfer functions for most Kinematics sensors and recorders.

The sensors' transfer functions are based on theoretical models of the electro-mechanical system. Equations are also provided for using the instrument calibration parameters to determine a model with a better "fit" for the unit you are using.

For digitizers with analog anti-alias filters we have provided the theoretical transfer functions for the designs based on the nominal component values at room temperature. There will be small variations in actual field conditions due to both component tolerances and the effect of temperature changes on the components. However, for most applications these errors are insignificant.

Altus Digitizers use Digital Filtering for anti-alias protection that is implemented as Finite Impulse Response (FIR) Filters. These filters are identical in each unit, as they are not subject to component tolerances. However, be sure that you have the coefficients for the software version installed in the units. This report contains the coefficients for current Altus software as of the date of this release. In order to provide the very steep "brickwall" response of the Altus these filters have several hundred taps. Thus, this information is provided for those who wish to do complex post processing. However, we do not recommend trying to increase the frequency range of the instrument by "boosting" the amplitude at the FIR filter's corner.

Kinematics World Wide Web site is a good source for updated information on the Transfer functions of new instruments or on the current coefficients used in Altus instruments' Digital Filters.

2. FBA-11/FBA 23/FBA-23DH Strong Motion Accelerometers

2.1. Description of the Dynamic System

Kinematics FBA accelerometers are closed-loop, force-feedback sensors measuring the relative displacement of moving mass with respect to the sensor case. Their voltage output is proportional to the input acceleration in the frequency band from DC to 50 Hz (optionally to 100 Hz). The sensor's properties are essentially equivalent to

a second-order dynamic system with one pair of complex conjugate poles. The sensors' transfer function (TF) depends almost entirely on the electronic components rather than on the mechanical components of the sensors. The influence on the Transfer Function of the physical mass, mechanical damping, spring elements and internal RC low-pass filter in the trans-conductance amplifier stage within the closed-loop path of the sensor are negligible for most applications. For more accurate transfer functions at high frequencies above 20 Hz, you can take into account the additional pole of a passive, low-pass RC filter in the post-amplifier stage of the sensor electronics.

The sensors are factory-calibrated as second-order systems. Nominal resonant frequency is 50 Hz (or 100 Hz or 90 Hz for a 4 g unit) and nominal relative damping is 0.707. Resonant frequency, relative damping, and static gain are factory-adjusted close to the nominal values and are noted in the calibration sheet of each sensor. In order to be more precise, you must take into account poles based on values from the calibration sheet rather than the nominal values. Sensors with different absolute sensitivity have the same second-order poles. However, the low-pass filter in the post-amplifier stage depends on the sensor's full-scale sensitivity. Poles pertaining to mechanical elements and the internal low-pass filter in the trans-conductance amplifier (which are insignificant in most cases) also depend on the full-scale sensitivity of the sensors.

If you wish, you can determine the modification of the TF due to mechanical elements and the internal low-pass filter. An identification program like UNICAL based on the inversion of step or impulse response or MatLab functions can be used for this purpose.

2.2. Theoretical Second-Order Transfer Function

The theoretical second-order Transfer Function of the FBA is:

1

$$\frac{E_{OUT}}{E_{DC}} = \frac{\omega_n^2}{s^2 + 2 \cdot \zeta \cdot \omega_n \cdot s + \omega_n^2}$$

where ω_n is the natural frequency
 s is the Laplace operator
 ζ is the relative damping
 E_{OUT} is the output voltage
 E_{DC} is the sensitivity

This can be solved as a quadratic to find the two complex poles:

2

$$p_1 = -\zeta \cdot \omega_n + j \cdot \omega_n \cdot \sqrt{1 - \zeta^2}$$

$$p_2 = -\zeta \cdot \omega_n - j \cdot \omega_n \cdot \sqrt{1 - \zeta^2}$$

These equations can then be used with the calibration constants below or those taken from the unit's calibration data to calculate these poles.

2.3. Absolute Gain

Kinometrics FBA sensors are produced in 0.25, 0.5, 1, 2, and 4 g full-scale sensitivity. This corresponds to the following nominal absolute sensitivity at DC.

Full Scale Range /g's	Absolute Gain Vs ² /m
0.25g	1.0190
0.5g	0.5095
1.0g	0.2550
2.0g	0.1270
4.0g	0.0637

Accurate calibration values for each sensor are given on the sensor's calibration card.

2.4. Practical Damping & Natural Frequency

The Calibration Card for the FBA also contains the Natural Frequency and damping of the sensor. Thus, to most precisely determine the poles and zeroes these values can be substituted into equations 2 and 3.

2.5. Second-Order Transfer Function Poles for Nominal FBA Parameters

If we assume that the FBA's natural frequency is 50, 90, or 100 Hz, and the damping is precisely 0.707 we can obtain the following nominal poles for the second-order representation for constant input acceleration:

FBA Natural Frequency /Hz	Pole P1		Pole P2	
	Real Component [rad/s]	Imaginary Component [rad/s]	Real Component [rad/s]	Imaginary Component [rad/s]
50	- 222.1	+ 222.1	- 222.1	- 222.1
90	- 399.8	+ 399.8	- 399.8	- 399.8
100	- 444.2	+ 444.2	- 444.2	- 444.2

We can get a further improvement in the accuracy of the model by considering the pole due to the low-pass RC filter in the post amplifier. This pole is dependent on the Natural Frequency and range of the sensors.

FBA Natural Frequency /Hz	FBA Range /g	Pole P3	
		Real Component [rad/s]	Imaginary Component [rad/s]
50, 100	0.25, 0.5, 1	- 1000	0
50, 90	2,4	- 1500	0

Now as an example of the full TF, a 50 Hz, 1g FBA would have the following nominal poles:

	FBA Natural Frequency [Hz]	Pole P1	Pole P2	Pole P3
Real Component [rad/s]	50	-222.1	- 222.1	-1000
Imaginary Component [rad/s]		222.1	-222.1	0

Note: To get the TF representation reduced to a constant input displacement compatible with the standardized GSE PAZ calibration header (CAL1 format), two zeros at the coordinate origin of the s-plane must be added to the above transfer functions.

	FBA Natural Frequency [Hz] and full scale sensitivity [g]	Zero Z1	Zero Z2
Real Component [rad/s]	All	0	0
Imaginary Component [rad/s]		0	0

3. The SS-1 Short-Period Seismometer

3.1. Description of the Dynamic System

The Ranger SS-1 short-period seismometer is a passive electrodynamic seismic sensor. Its output above the resonant frequency is essentially proportional to the ground velocity. It can be accurately represented by a second-order dynamic system. Its nominal resonant frequency is 1 Hz. The sensor's damping "element" is electrodynamic in nature. The choice of the damping is left to you. The damping is determined by selection of an external damping resistor. Normally, a nominal damping of around 0.7 is used.

SS-1 seismometers are completely calibrated in our factory. Resonant frequency and absolute sensitivity are adjusted close to nominal values. The open generator

constant, G_0 , of the main coil, the external damping resistor, R_x , for nominal relative damping of 0.7, and the internal coil resistance, R_c , are shown on the calibration sheet of each sensor. After selection of the damping resistor, coil resistance allows calculation of the loaded generator constant, G_L .

Note: If you calibrate the SS-1 using the calibration coil the resultant transfer function will include the affect of the mutual induction between the calibration coil and the main coil. This becomes significant for frequencies above 5 Hz. This interaction can be removed by post-processing to account for the mutual inductance.

3.2. Absolute Gain, Nominal Zeros and Poles of Transfer Function

The nominal open-generator constant G_0 of the SS-1 for standard coil resistance, R_c , of 5000 ohms is

$$G_0 = 345 \text{ [Vs/m]}$$

For practical use, the loaded generator constant G_L must be calculated according to the equation below, where G_0 is the open generator constant, R_c is the coil resistance given in the calibration sheet, and R_x is the damping resistor you select.

$$G_L = G_0 \cdot \frac{R_x}{R_x + R_c}$$

Zeros and poles of the nominal transfer function reduced to the constant input velocity are:

	Pole P1	Pole P2	Zero Z1	Zero Z2
Real part [rad/s]	- 4.44	- 4.44	0	0
Imaginary part [rad/s]	+ 4.44	- 4.44	0	0

You may use equations 2 and 3, and data from the sensor's calibration sheet for a more precise pole zero representation.

Note: To get the TF representation reduced to a constant input displacement compatible with the standardized GSE PAZ calibration header (CAL1 format), one zero at the coordinate origin of the s plane must be added to the above transfer function.

	Zero Z3
Real part [rad/s]	0
Imaginary part [rad/s]	0

4. WR1 Wide Band Seismometer

4.1. Description of the Dynamic System

The WR-1 wide-band seismometer is mechanically similar to the SS-1 seismometer; however, its mass position is controlled by electronic circuitry in a closed-loop, force-feedback manner. Its closed-loop TF has the properties of a second-order system with a corner frequency at 20 Hz and relative damping close to a nominal value 0.707. This corner frequency is determined almost entirely by the electronic components. The sensor has three outputs:

- DC-coupled acceleration output. Acceleration-proportional output with frequency pass-band from DC to 20 Hz
- AC-coupled acceleration output. Acceleration-proportional output with frequency pass-band from 0.05 Hz to 20 Hz (optionally from 0.02 Hz to 20 Hz), and
- Velocity output. Velocity-proportional output with frequency pass-band from 0.05 Hz to 20 Hz (optionally from 0.02 Hz to 20 Hz.)

The sensor is composed of a force-feedback loop that measures relative displacement of moving mass with respect to the case of the sensor with a capacitive displacement transducer. The output of this stage is proportional to the input acceleration. The second stage is a passive, single-pole, RC low-pass filter that is followed by a post-amplifier that also contains an additional single-pole, low-pass filter.

The output of this amplifier is the DC-coupled acceleration-proportional output of the WR-1. This signal is then fed through a passive, single-pole, high-pass filter and an active signal follower. This stage produces the band-pass limited, AC-coupled, acceleration-proportional output of the sensor. This signal is then fed through an active integrator, whose output is proportional to ground velocity in the frequency range from 0.05 (optionally from 0.02) to 20 Hz. The integrator's electronics has two poles and one zero. This zero and the higher pole lay at approximately the same frequency, thus this stage effectively integrates all input signals above the lower pole.

4.2. Acceleration-Proportional DC-Coupled Output -- Absolute Gain and Transfer Function

The DC-coupled acceleration-proportional output of the WR-1 has the following nominal generator constant at DC:

$$G_0 = 25.5 \text{ [Vs}^2\text{/m]} \text{ or } 250 \text{ [V/g]}$$

The precise value for each sensor is given on its calibration sheet.

Since the WR-1 is an active sensor, there is no need for a damping resistor or for calculation of the loaded generator constant for each application.

The poles of the nominal transfer function for DC-coupled acceleration-proportional output is:

	Pole P1	Pole P2	Pole P3	Pole P4
Real part [rad/s]	- 88.8	- 88.8	- 1000	- 1030
Imaginary part [rad/s]	+ 88.8	- 88.8	0	0

Note: To get the TF representation reduced to a constant input displacement compatible with the standardized GSE PAZ calibration header (CAL1 format), two zeros at the coordinate origin of the s-plane must be added to the above transfer functions.

	Zero Z1	Zero Z2
Real Component [rad/s]	0	0
Imaginary Component [rad/s]	0	0

4.3. Acceleration-Proportional AC-Coupled Output -- Absolute Gain and Transfer Function

The WR-1 seismometer has the following nominal generator constant at the AC-coupled acceleration-proportional output and at 1 Hz:

$$G_0 = 25.5 \text{ [Vs}^2\text{/m]} \text{ or } 250 \text{ [V/g]}$$

The precise value for each sensor is shown on its calibration sheet. Since the WR-1 is an active sensor, there is no need for a damping resistor or for calculation of the loaded generator constant for each application.

Zeros and poles of nominal transfer function for AC-coupled acceleration output are as follows:

Standard WR-1 with a 0.05 Hz low frequency corner:

	Pole P1	Pole P2	Pole P3	Pole P4	Pole P5	Zero Z1
Real part [rad/s]	- 88.8	- 88.8	- 1000	- 1030	- 0.314	0
Imaginary part [rad/s]	+ 88.8	- 88.8	0	0	0	0

The WR-1 with a 0.02 Hz low frequency corner option:

	Pole P1	Pole P2	Pole P3	Pole P4	Pole P5	Zero Z1
Real part [rad/s]	- 88.8	- 88.8	- 1000	- 1030	- 0.126	0
Imaginary part [rad/s]	+ 88.8	- 88.8	0	0	0	0

Note: To get the TF representation reduced to a constant input displacement compatible with the standardized GSE PAZ calibration header (CAL1 format), two zeros at the coordinate origin of the s-plane must be added to the above transfer functions.

	Zero Z2	Zero Z3
Real Component [rad/s]	0	0
Imaginary Component [rad/s]	0	0

4.4. Velocity-Proportional Output -- Absolute Gain and Transfer Function

The WR-1 seismometer has the following nominal generator constant at the velocity proportional output and 1 Hz:

$$G_0 = 160 \text{ [Vs/m]}$$

The precise value for each sensor is shown on its calibration sheet. Since the WR-1 is an active sensor, there is no need for a damping resistor or for calculation of the loaded generator constant.

Zeros and poles of the nominal transfer function of velocity-proportional output are as follows:

A standard WR-1 with a 0.05 low frequency corner:

	Pole P1	Pole P2	Pole P3	Pole P4	Pole P5	Pole P6	Pole P7	Zero Z1	Zero Z2	Zero Z3
Real part [rad/s]	- 88.8	- 88.8	- 1000	- 1030	- 0.314	- 6.45	- 0.30	0	- 7.25	0
Imaginary part [rad/s]	+ 88.8	- 88.8	0	0	0	0	0	0	0	0

The WR-1 with a 0.02 low frequency corner option:

	Pole P1	Pole P2	Pole P3	Pole P4	Pole P5	Pole P6	Pole P7	Zero Z1	Zero Z2	Zero Z3
Real part [rad/s]	- 88.8	- 88.8	- 1000	- 1030	- 0.126	- 6.45	- 0.098	0	- 7.25	0
Imaginary part [rad/s]	+ 88.8	- 88.8	0	0	0	0	0	0	0	0

Note: To get the TF representation reduced to a constant input displacement compatible with the standardized GSE PAZ calibration header (CAL1 format), one zero at the coordinate origin of the s-plane must be added to the above transfer functions.

	Zero Z4
Real Component [rad/s]	0
Imaginary Component [rad/s]	0

5. The SSA-1 and SSA-2 Digital Accelerometers

The digital SSA-1 and SSA-2 accelerographs have standard 50 Hz FBA sensors built-in. All zeros and poles of the FBA are also active in the SSA-1's transfer function. In addition there is an analog two-pole Butterworth anti-aliasing filter at 50 Hz built into both accelerographs. Thus, the overall TF of an SSA-1 or SSA-2 with 50 Hz, 2 g full-scale, sensitivity-internal accelerometer has the following zeros and poles:

	Pole P1	Pole P2	Pole P3	Pole P4	Pole P5
Real part [rad/s]	- 222.1	- 222.1	- 1500	- 222.1	- 222.1
Imaginary part [rad/s]	+ 222.1	- 222.1	0	+ 222.1	- 222.1

The overall TF of an SSA-1 or SSA-2 with 50 Hz, 0.25, 0.5, or 1 g full-scale sensitivity-internal accelerometer has the following zeros and poles:

	Pole P1	Pole P2	Pole P3	Pole P4	Pole P5
Real part [rad/s]	- 222.1	- 222.1	- 1000	- 222.1	- 222.1
Imaginary part [rad/s]	+ 222.1	- 222.1	0	+ 222.1	- 222.1

For accelerographs with external sensors you must add pole p4 and p5 to the zeros and poles of the actual sensor to get the full transfer function of the recording system.

Note: To get the TF representation reduced to constant input displacement compatible with the standardized GSE PAZ calibration header (CAL1 format), two zeros at the coordinate origin of the s-plane must be added to the above transfer function.

	Zero Z1	Zero Z2
Real part [rad/s]	0	0
Imaginary Part [rad/s]	0	0

6. SSA-16 Digital Accelerometer

The SSA-16 digital accelerometer has standard FBA-11 sensors built-in. All zeros and poles of the FBA are also active in the SSA-16's transfer function. In addition there is an analog 6-pole Butterworth anti-aliasing filter at 50 Hz built into the SSA-16 accelerometer. Thus the overall TF of the SSA-16 with 50 Hz, 2 g full-scale sensitivity-internal accelerometer has the following zeros and poles:

	Pole P1	Pole P2	Pole P3	Pole P4	Pole P5	Pole P6	Pole P7	Pole P8	Pole P9
Re. part [rad/s]	- 222.1	- 222.1	- 1500	- 81.3	- 81.3	- 222.1	- 222.1	- 303.5	- 303.5
Im. part [rad/s]	+ 222.1	- 222.1	0	+ 303.5	- 303.5	+ 222.1	- 222.1	+ 81.3	- 81.3

If another type of accelerometer is installed in the SSA-16, take into account its transfer function and add poles from P4 to P9 from the above table.

Note: To get the TF representation reduced to constant input displacement compatible with the standardized GSE PAZ calibration header (CAL1 format), two zeros at the coordinate origin of the s plane

	Zero Z1	Zero Z2
Real Part [rad/s]	0	0
Imaginary Part [rad/s]	0	0

must be added to the above transfer function.

7. SSR-1 Digital Recorder

The SSR-1 digital recorder uses analog, six-pole anti-aliasing filters. Their corner frequency depends on the options you chose. The following options for corner frequencies are available: 1, 2.5, 5, 10, 15, 25, 50, 125, 250 Hz. Standard values are 50, 15 and 5 Hz. You may order either Butterworth or Bessel filters. Three filters can be built into the recorder at the same time, and they are software selectable.

The SSR-1 recorder also has a built-in, first-order RC high-pass filter at a 0.01 Hz corner frequency. It can be switched on or off through software.

The following table shows the poles of standard corner frequencies and Butterworth SSR-1's anti-aliasing filters.

50 Hz Butterworth antialiasing filter:

	Pole P1	Pole P2	Pole P3	Pole P4	Pole P5	Pole P6
Real part [rad/s]	- 81.3	- 81.3	-222.1	-222.1	-303.5	-303.5
Imaginary part [rad/s]	+ 303.5	- 303.5	+ 222.1	- 222.1	+ 81.3	- 81.3

15 Hz Butterworth antialiasing filter:

	Pole P1	Pole P2	Pole P3	Pole P4	Pole P5	Pole P6
Real part [rad/s]	- 24.4	- 24.4	- 66.6	- 66.6	- 91.0	- 91.0
Imaginary part [rad/s]	+ 91.0	- 91.0	+ 66.6	- 66.6	+ 24.4	- 24.4

5 Hz Butterworth antialiasing filter:

	Pole P1	Pole P2	Pole P3	Pole P4	Pole P5	Pole P6
Real part [rad/s]	- 8.13	- 8.13	- 22.2	- 22.2	- 30.4	- 30.4
Imaginary part [rad/s]	+ 30.4	- 30.4	+ 22.2	- 22.2	+ 8.13	- 8.13

The general case for a Butterworth filter with corner frequency, f_c [Hz]:

$$p_1[\text{rad} / \text{s}] = -2\pi \cdot f[\text{Hz}](0.259 + j0.966)$$

$$p_2[\text{rad} / \text{s}] = -2\pi \cdot f[\text{Hz}](0.259 - j0.966)$$

$$p_3[\text{rad} / \text{s}] = -2\pi \cdot f[\text{Hz}](0.707 + j0.707)$$

$$p_4[\text{rad} / \text{s}] = -2\pi \cdot f[\text{Hz}](0.707 - j0.707)$$

$$p_5[\text{rad} / \text{s}] = -2\pi \cdot f[\text{Hz}](0.966 + j0.259)$$

$$p_6[\text{rad} / \text{s}] = -2\pi \cdot f[\text{Hz}](0.966 - j0.259)$$

Zeros and poles of standard corner frequencies and Bessel anti-aliasing filters are shown below:

50 Hz Bessel antialiasing filter:

	Pole P1	Pole P2	Pole P3	Pole P4	Pole P5	Pole P6
Real part [rad/s]	-169.2	-169.2	-251.2	-251.2	-285.7	-285.7
Imaginary part [rad/s]	302.1	-302.1	176.6	-176.6	58.3	-58.3

15 Hz Bessel antialiasing filter:

	Pole P1	Pole P2	Pole P3	Pole P4	Pole P5	Pole P6
Real part [rad/s]	-50.8	-50.8	-75.4	-75.4	-85.7	-85.7
Imaginary part [rad/s]	90.6	-90.6	53.0	-53.0	17.5	-17.5

5 Hz Bessel antialiasing filter:

	Pole P1	Pole P2	Pole P3	Pole P4	Pole P5	Pole P6
Real part [rad/s]	-16.9	-16.9	-25.1	-25.1	-28.6	-28.6
Imaginary part [rad/s]	30.2	-30.2	17.7	-17.7	5.83	-5.83

The zero and pole of the high-pass filter with the corner frequency at 0.01 [Hz] (only active if switched on) is shown below:

	Pole P7	Zero Z1
Real Part rad/s]	- 0.0628	0
Imaginary Part [rad/s]	0	0

Zeros and poles of the SSR-1 data recorder are simply added to the transfer function of the sensors used, no matter which kind of representation of the TF we deal with.

8. The Altus Family of Seismic Recorders

The Altus family of seismic recorders, the K2, Mt. Whitney, Makalu and Etna/EtnaSI use digital filters to provide anti-alias protection. These filters are implemented as two or three stage, multi-rate, Finite Impulse Response (FIR) filters. They offer extremely steep low pass amplitude response. The -3dB point is at 40% of the Sampling Frequency, while at the Nyquist Frequency (50% of the Sampling Frequency) the amplitude is -120dB. In all but the Etnas you can select either a causal or a non-causal version of the final filter depending on your application.

The first-stage (A) filters are non-causal FIR filters. The Etnas require that their first filter have sinc³ compensation. A 2KHz data stream from the A/D converter is decimated by the appropriate factor to get an intermediate data stream at twice the final sampling rate. At final sample rates below 100 sps, this requires a second A filter. The final decimate by 2 brickwall filter is available with either causal or non-causal coefficients as the application requires. The Makalu has double precision coefficients. The table below lists the recorder, filter file name, number of coefficients and decimation for the anti-alias filters used in the Altus Recorder Family. All coefficients are contained in the accompanying .COE files.

Altus Recorder Anti-Alias Filters			
K2 and Mt. Whitney			
Final sps	First A Filter	Second A Filter	Final Brickwall
250	FIRA250 (47, /4)	---	FIRBNC (137, /2) FIRBC (111, /2)
200	FIRA200 (57, /5)	---	“”
100	FIRA100 (113, /10)	---	“”
50	FIRA100 (113, /10)	FIRA50 (59, /2)	“”
40	FIRA200 (57, /5)	FIRA200 (57, /5)	“”
20	FIRA100 (113, /10)	FIRA200 (57, /5)	“”
Makalu			
Final sps	First A Filter	Second A Filter	Final Brickwall
250	FIRA250DP (57, /4)		FIRBNCDP (173, /2) FIRBCDP (173, /2)
200	FIRA200DP (71, /5)	---	“”
100	FIRA100DP (143, /10)	---	“”
50	FIRA100DP (143, /10)	FIRA50DP (85, /2)	“”
	FIRA200DP (71, /5)	FIRA200DP (71, /5)	“”
	FIRA100DP (143, /10)	FIRA200DP (71, /10)	“”
Etna			
Final sps	First A Filter	Second A Filter	Final Brickwall
250	FIRA250SS3 (47, /4)	---	FIRBNC (137, /2)
200	FIRA200S3 (57, /5)	---	“”
100	FIRA100S3 (113, /10)	---	“”